# MATH 118: Midterm 2

Name: _____

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

| Problem | Score | Points |
|---|---|---|
| 1 | | 10 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| | | **60** |

1. Short answer questions:

   (a) Given three functions

   $$f(x) = \sqrt{x-2}, \qquad g(x) = \frac{2x}{3x+1}, \qquad h(x) = x^4$$

   find the composition $g \circ f \circ h$.

   (b) Given a polynomial $f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$, should $f(4.5)$ be positive or negative and why?

   (c) Find a degree four polynomial with zeros $i\sqrt{3}$ and $5i$.

   (d) Given a base function $f(x) = \sqrt{x}$ and two transformed functions

   $$g(x) = \sqrt{x-2} \qquad\qquad h(x) = \sqrt{\frac{1}{2}x - 2}$$

   do both $g(x)$ and $h(x)$ have the same horizontal shift from $f(x)$? If not, state both of the horizontal shifts of $g(x)$ and $h(x)$.

2. Suppose $f(x) = x^2 - x$.

   (a) What is the domain of $f(x)$?

   (b) Find a complete factorization of $f(x)$.

   (c) Calculate and **fully expand + simplify** the expression $\dfrac{f(x+h) - f(x)}{h}$.

3. Given the polynomial $P(x) = x^4 + 4x^3 + 5x^2 + 4x + 4$:

   (a) What is the average rate of change of $P(x)$ on $[0, 1]$?

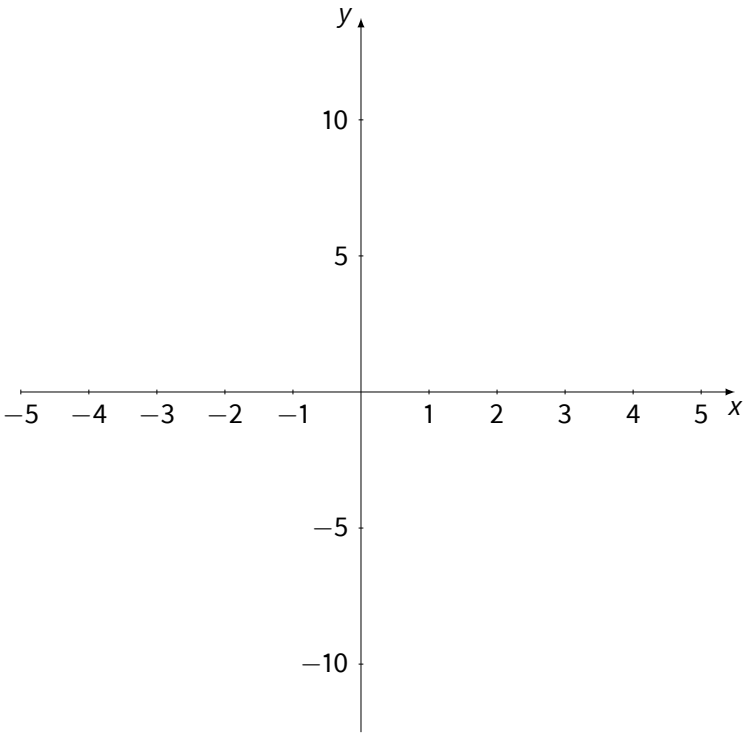
   (b) Is $P(x)$ even, odd, or neither? Full credit requires using the definition of even/odd.

   (c) $x = -2$ is a zero of multiplicity two for $P(x)$. Use this information to find a complete factorization of $P(x)$.

4. Sketch an accurate graph of the polynomial

$$P(x) = x^3 - x^2 - 6x$$

using the four step process.



5. Given the function

$$f(x) = 4 - x^2, \ x \geq 0$$

(a) Calculate the inverse $f^{-1}$ algebraically.

(b) Use the Inverse Function Property to verify your result of $f^{-1}$ is actually the inverse of $f(x)$.

6. Given

$$F(x) = \sqrt{x-1} \qquad G(x) = -(x^2 - 1)$$

(a) Find the domain of $F(x)$.

(b) Decompose $F(x)$ into two functions $f$ and $g$ where $f \circ g = F$. You are not allowed to choose $f(x) = x$ or $g(x) = x$.

(c) Calculate $(F \circ G)(0)$.

(d) Find the function $F \circ G$ and explain why the domain of this function is the single number $x = 0$.